Robert M. Briggs,
1412 W. 35th St., Apt. 17,
Phoenix,
Arizona

May 13, 1964

Dear Franklin and Gertrude,

The other evening the topic of ~~conver~~ conversation with the Hamiltons ~~was~~ was transcendental number. It was apparent from this conversation that we needed more information and if you have time, could you answer the following questions for us please.

(1) Where did the term "Transcendental" orginate and by whom?

(2) Are transcendental numbers irrational numbers or are they excluded from that group?

(3) If transcendental numbers are irrational numbers, then how can there be more transcendental numbers than rational numbers?

(4) What significance ~~does~~ do transcendental numbers have if any?

(5) Is mathematics of today basically a rediscovery of past knowledge, or ~~is~~ is it a creation of expression by the present race?

Erma has become very interested in transcendental numbers and their meaning. Any information on transcendental numbers would be greatly appreciated.

The last couple of weeks I have been getting up with the sun and I find it quite stimulating. My mind is very clear at this time of day so I have been reading occult literature before going to work. Before I started using this procedure, I would come home after 11 hours of work and would be too tired to read & comprehend any occult literature.

I will not receive a vacation this year so I will not be able to come to the ranch this summer.

I sure want to thank you for sending pictures of Jerome to me. It is alway nice to have pictures of the town you were born in.

Tell Peter & Peggy hello for me.

Love,

Bob

P.O. Box 559,
Lone Pine, Calif.,
May 15, 1964.

Dear Bob:

Somehow my first impression upon reading your letter was that you had in mind "Transfinite Numbers" and I soon realized an adequate answer would be almost a book, if the ramifications in Mysticism and Philosophy along with Mathematics was taken into account. However, the error was not so far-fetched when one remembers that Cantor's proof of the existence of a non-denumerable Transfinite involved the Transcendental Numbers.

To determine what Transcendental Numbers are it is desirable to begin with the definition of Algebraic Numbers. For a first approximation we may say: "An algebraic number is any number that can be a root of an algebraic equation having integral coefficients." But this becomes extended in the light of Cantor's proof that any root of an algebraic equation having algebraic numbers (not exclusively integers) as coefficients is an algebraic number. An algebraic equation is one in which trigonometric, exponential and logarithmic functions do not appear. This extends the notion of algebraic number to all the ordinary familiar numbers except two, i.e., all integers, both positve and negative, all fractions, all irrationals of the type such as the nth root of a, all imaginaries and all complex numbers of the form a plus ib, where i is the square roor of minus 1. The two excluded familiar numbers are pi and e.

Without going into the question of whether the principal of the excluded middle, characteristic of Aristotelian logic, in reasoning about infinite classes is valid or not, Cantor proved or seemed to prove that the ~~xxxx~~ sum-total of all algebraic numbers is denumerable, i.e., can be placed in one-to-one correlation with the positive integers. That is to say that they can be counted, though it would take an infinite time to do so. Quite otherwise is it with the Transcendental Numbers. Cantor's demonstration seems to show that they are so much more numerous than algebraic numbers that they cannot be counted, in other words they are non-denumerable. They belong to an infinity of a higher order such as $\omega\omega^{\omega}$.

Assuming that the Cantor proof that the cardinality of the Transcendental Numbers is infinitely greater than the cardinality of all ~~other~~ algebraic numbers combined, it comes with something of a shock to realize that only two Transcendentals are well known. In the modern period of mathematics (i.e., from DesCartes to the present) a number of classes of Transcendental Numbers have been discovered, of which the following are two examples:

1. Numbers of the form $\frac{1}{n}$ plus $\frac{1}{n^2}$ plus $\frac{1}{n^6}$ plus $\frac{1}{n^{24}}$ plus ------
   where n is a real number greater than 1 and the indecies are the series of factorals.

2. All numbers of the type a^b where a is neither 0 nor 1 and b is any irrational algebraic number.

It is easily seen that in either case this gives us an infinity of Transcendental numbers. The history of mathematice reveals that it is easier to find infinite classes of Transcendentals than to prove that any given number, such as, pi and e, are Transcendentals. Hermite gave the proof in the case of e . The transcendentality of e was proven by Hermite and of pi by Lindermann.

The numbers pi and e are of such importance that someone has said a universe without these numbers could not exist, though this view is not generally accepted. But Kasner and Newman in their book, "Mathematics and the Imagination" have said, "But without these mathematical artifacts, what we know about the sun and tides, indeed our ability to describe all natural phenomena, physical, biological, chemical or statistical, would be reduced to primitive dimensions." The question here is really philosophical. As an Introceptional Idealist I lean toward a positive answer to the question: Is a necessity for thought a necessity for Reality?

(The last statement needs amplification. No doubt, on the ordinary level of thought, such as that which characterizes empiric science, there is a contrast between the concept or idea and its referent. Thus the concept "tree" and the tree itself belong to two different orders, or, at least, so they appear. Here it would not seem that a law of thought is a law of nature. It all appears dualistic. But even at a lower level of the High Indifference, as reported in "Pathways", concept and the object of conception fused into Identity. The dichotomy vanished. In fact this is an aspect of advaita. At this level one would say that there is no difference between a law of thought and a law of Reality. Now all this is pertenent to the mathematical question if one has in mind my theory of the nature of mathematics, particulary in the case of Transcendental Numbers and Transfinite Numbers. Bear in mind we are speaking only of Pure Mathematics, not applied mathematics, which is only the science of allowed error. First, there are three recognized theories of the nature of mathematics, i.e., Formalism, led by Hilbert, Logisticism, led by Russell, and Intuitionalism, led by Brouwer and Weyl. The formalists would say that mathematics is no more than a sort of meaningless game of chess wh~~e~~ihh, none the less, is played assiducusly. The logisticists would maintain that mathematics is entirely reducible to logic, though a more comprehensive logic than that of Aristotle. The intuitionalists would maintain that only some of mathema~~tt~~es is valid, and they would abandon all mathematics involving the infinite. This comes closest to the engineer's point of view. I do not find any of these views adequate, though, no doubt, reflecting part truths. Thus, no doubt mathematics is logic, but it is also more than logic. Spengler has well pointed out that at its highest ranges, the mathematician transcends abstraction and logic and is guided by Vision. Logic alone is only a critic, it does not create or discover. This leads to my theory.

(I affirm what might be called the Gnostic ~~Teery~~ Theory or the thesis that mathematics is the one body of knowledge which has descended from the Gnosis into the Maya (or Sangsara) with no, or, at least, minimal corruption. Thus it is the greatest purity we know short of liberating Realization. It is thus the one facet of Buddha (Enlightenment) which remains unveiled in the Sangsara. It is therefore the Yogic thread par excellence, and so I found it in 1936. Here the thesis would be that authentic mathematics deals only with the infinite, veiled or explicit, and never with special cases.)

Returning to the main thread of our discussion, it will be desirable to show some of the reason for the importance of e and pi. Pi was known by the ancients and played a part in the deeper phases of religious Initiation. Piazzi Smythe maintains that the Great Pyrimid was built as a monument to pi, wherein it is revealed correct to five decimal places. The word "Elohim" when the numerical values of the Hebrew letters is taken is found to be an anagram of pi. Elohim is equivalent to Kumara, Dhyan Chohan, Tathagata and, I believe, the Supramental Being of Sri Aurobindo. These Beings appear to stand on the border between the truly Transcendent and the mundane. Here we get a hint as to the real meaning of the squaring of the circle, or rather the circularizing of the square. (In the lower sense the circle cannot be squared if we are restriced to the use of a straight-edge and the ~~xxxxxx~~ compass, but can be done by other means. Pi is the key to this process) The circle represents the Transcendent the square the mundane. It bears very definitely on the Yogic problem. (In my Mandala the transcendent of the circle is in its tarn transcended by the equilateral hyperbola.) But pi in relation to the circle is only part of its significance.

Pi is also the value of certain non-terminating series, products and fractions, as follows:

$$\frac{\pi}{2} = \frac{2}{1} \times \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \frac{8}{9} \times \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \cdots$$

$$\frac{\pi}{4} = 4\left(\frac{1}{5} - \frac{1}{3\cdot 5^3} + \frac{1}{5\cdot 5^5} - \frac{1}{7\cdot 5^7} + \cdots\right) - \left(\frac{1}{239} - \frac{1}{3\cdot 239^3} + \frac{1}{5\cdot 239^5} - \frac{1}{7\cdot 239^7} + \cdots\right)$$

$$\pi = \cfrac{4}{1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{\cdots}}}}}$$

Finally pi has a place in the Theory of Probability . On a flat surfae of conveninet size draw a series of parallel lines at distance a from each other. Provide yourself with needles of length a. Throw these needles so that they land at random on the board. Record in two columns the number that lie across the lines and the number that lie in the spaces between the lines. Do this a large number of times. The ratio will approach the value of 2/pi. 3,408 tosses made in 1901 gave the following value for pi, i.e., 3.1415929. I find this exceedingly significant. It means that what we call random or chance is governed by law and, therefore, there is no such thing as real randomness or chance. Thus randomness and chance are not facts in nature but a manifestation of our ignorance. Applying this to the statistical theory of physical law, which Weiner has shown to be currant today, our laws are not giving realities of Nature, but are expressions of the limitations of our knowledge.

e or $L_{n \to \infty} (1 + \frac{1}{N})^n$ seems to be a number belonging to the modern period (i.e., since Des Cartes). Euler suggested e as the symbol for this number. First of all it is highly important in the field of logarithms. It is the base of the Natural or Napier system of logarithms and is by far the most important. (I can understand why you should be prejudiced in favor of the Briggs system.) But beyond this, like pi, it is the value of different infinite series, non-terminating fractions and products. Still more interesting is the fact that this number enters into all formulae related to living processes, such as, plants, animals, humans, corporations, nations and all human institutions. You know its importance in calculus and its unique property of being its own derivative. Thus $d/dx\ e^x$ equals e^x. One interpretation of this is that rate of growth is proportional to state of growth.

If these two Transcendental Numbers are so important one is inclined to wonder about the non-denumerable infinity of other Transcendentals. The character of all these is more apt to be revealed in their non-terminating formula forms than in their decimal forms. However you may like a look at pi and e in the decimal form, thus

pi equals 3.14159265358979323846----------
(Shanks carried this to 707 decimal places giving most of his life to the job.)
e equals 2.718281828459045235360287 4---------

(Incidentially, all numbers can be written in the non-terminating form, thus
1 equals .9999999999999999999999999-----to infinity.

Important for Cantor's proofs.)

Now to your questions:

1. Where did the term "Transcendental" origninate and by whom?

Apparently it was Hermite, a French mathematician. Though the mathematical definition is only technical, a number that canot be the root of an algebraic equation, yet I suspect an unconscious factor in the selection of the word. The tie-in with the Transfinite through Cantor, their non-denumerable infinity, and the importance of the two best known Transcendentals suggest this.

2. Are Transcendental numbers irrational numbers or are they excluded from that group?

I have seen them excluded, but in general they are in cluded. In the latter case they are to be distinguished from the ordinary irrationals that can be roots of algebraic equations, such as the nth root of a in all cases where this root is not exact.

3. If Transcendental Numbers are irrational numbers, then how can there be more Transcendental Numbers than rational numbers?

I suggest that you study Cantor's proofs.

4. What significance do Transcendental Numbers have if any?

I have already handled this in the case of pi and e. Beyond this, they demonstrate the existence of a non-denumerable Infinite if Cantor's proof is valid. And beyond this one stands in the presence of Mystery. (Refer to the first paragraph of the High Indifference in "Pathways" where I refer to a higher Infinity transcending a lower Infinity. Figures provided by the conception of the Transfinite numbers enabled me to be articulate in writing on the High Indifference beyond what would have been otherwise possible.)

5. Is mathematics of today basically a rediscovery of past knowledge or is it a creation of expression by thepresent race?

There may be some rediscovery, particularly in the earlier work of the Western Culture, but I think there is a good deal of pioneering. Senior was impressed and said He planned to take an incarnation to acquire this modern material. Your use of the word "creation" brings up a problem where there are philosophic differences. There are those who maintain that mathematics is only language and therefore invention. I hold the view that while mathematics of necessity has a language, and this no doubt is invented, more importantly it gives substantive Truth. The latter is discovered.

If you have trouble with this you may get help on the mathematics from Mr. Hamilton, on the Philosophy from the head of the Philosophy Dept., and on the veiled Theosophical references from Erma.

I hope you do not ask about the Transfinite for that might take a book, and I do not feel that energetic.

Good luck,